การศึกษาผลต่อการเกิดแนวต้านการสูญเสียพลังงานและอนุภาคพลาสมา อันเนื่องจากฟลักซ์ความร้อนและความไม่เสถียรของฟลักซ์ โดยใช้โมเดลแบบไบเฟอร์เคชัน

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บทคัดย่อ

งานวิจัยนี้เป็นการศึกษาการเกิดของแนวด้านการสูญเสียพลังงานและอนุภาคพลาสมา สำหรับนิวเคลียร์ พิวชัน โดยใช้โมเดลไบเฟอร์เคชันอย่างง่ายที่มีสนามเดียว โมเดลนี้ประกอบไปด้วยกราฟรูปตัวเอส โดยมีส่วนเสถียร สองส่วนแสดงถึงพลาสมาในสภาวะประสิทธิภาพค่ำและสูง เชื่อมด้วยส่วนที่ไม่เสถียรคั่นตรงกลาง สมมติฐานที่ใช้ ในงานวิจัยได้แก่ ค่าความต่างของความเร็วในแนวโพลอยดัล v'_e เป็นตัวที่ไปลดการส่งผ่านพลังงานและอนุภาค แบบอะโนมอลัส โดยกำหนดให้ค่านี้แปลผันโดยตรงกับการเปลี่ยนแปลงของความดัน ในส่วนของงาน จะใช้วิธีการ ทางวิเคราะห์และทางตัวเลขเพื่อหาค่าที่จำเป็นสำหรับการเกิดแนวด้านขึ้น อันได้แก่อัตราส่วนค่าสัมประสิทธิ์ของ การส่งผ่านแบบอะโนมอลัสหารด้วยแบบนีโอคลาสสิคอล และค่าการให้ฟลักซ์ความร้อน โดยในการวิเคราะห์จะใช้ แหล่งความร้อนในรูปแบบต่างๆ เช่น แบบค่าคงตัว แบบเกาซ์ และแบบไฮเปอร์โบลิคแทนเจนต์ อีกทั้งยังได้มี การศึกษาผลกระทบโดยตรงต่อการเกิดของแนวด้าน อันเนื่องมาจากความไม่เสถียรของ ฟลักซ์ความร้อน

คำสำคัญ : พลาสมา โทคาแมค การเปลี่ยนโหมดจากสภาวะต่ำไปสูง แนวต้านการสูญเสียพลังงานและอนุภาค พลาสมา

Study of Heat Flux Threshold and Perturbation Effect on Transport Barrier Formation Based on Bifurcation Model

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Abstract

Formation of transport barrier in fusion plasma is studied using a simple one-field bistable S-curve bifurcation model. This model is characterized by an S-line with two stable branches corresponding to the low (L) and high (H) confinement modes, connected by an unstable branch. Assumptions used in this model are such that the reduction in anomalous transport is caused by v'_E velocity shear effect and also this velocity shear is proportional to pressure gradient. In this study, analytical and numerical approaches are used to obtain necessary conditions for transport barrier formation, i.e. the ratio of anomalous over neoclassical coefficients and heat flux thresholds which must be exceeded. Several profiles of heat sources are considered in this work including constant, Gaussian, and hyperbolic tangent forms. Moreover, the effect of perturbation in heat flux is investigated with respect to transport barrier formation.

Keywords: plasmas, Tokamak, L-H transition, transport barrier

1. Introduction

Discovery of *H*-Mode in tokamak plasma is considered one of the most significant events in the history of fusion plasma research¹. When heat applied is high enough, the plasma makes a sudden transition from *L*-Mode (Low confinement) to *H*-Mode (High confinement). Consequently, the plasma performance is improved considerably including high temperature and long confinement time. It is known that the improved performance of *H*-mode results mainly from the formation of an edge transport barrier (ETB)² which is categorized by the steep in pressure or particle density gradient profile. An important goal of magnetic confinement fusion devices like tokamaks is to understand the mechanisms for the ETB formation or the so called L-H transition. The phenomenon of L-H transition has long been known to the fusion community. However the physics behind is still controversial. There are many approaches that attempting to explain the transition, including experimental and theoretical approaches ³. In this work, the simple S-curve bistable bifurcation model of one-field equation is used to explain the transition. It is based on fundamental physics in which the model is built from heat transport equation in tokamak plasma. The model is described by three branches, two of which are stable and correspond to L and H modes, while the last branch is unstable and corresponds to a prohibited regime. The suppression mechanism which leads to formation of ETB is due to velocity shear in poloidal direction v'_{E} . In this research, an analytical study is studied to find the threshold conditions for the transport barrier formation. Then a numerical code is used to study the dynamic of this transition including the heat flux and perturbation effect on L-H transition.

The paper is organized as follows: an introduction to the bifurcation model is given in section 2; an analytical study is shown in section 3, numerical results and discussions are presented in section 4; and the summary is discussed in section 5.

2. Bifurcation Model

Bifurcations in some quantity can take place when the equation which determines it has multi-valued or non-monotonic solutions as a function of an order variable, such as density or temperature gradients. The nature of the solutions allows a categorization of bifurcations. Figure 1 shows sample of bifurcation concepts when there exist multivalued solutions. The bifurcation model used in this work is mainly based on analytical work by Malkov and Diamond ⁴. The so-called bistable S-curve bifurcation model considers each mode (*L* and *H* modes) like a phase of plasma state so L-H transition resembles to a phase transition. In the model, the variation of system state like pressure or density gradients as a function of the control parameter such as heat or particle fluxes, respectively, is represented by an S-curve with two stable branches which correspond to the L and H modes. These two branches are connected by an unstable branch or transition region as shown in Fig. 1. Historically, the first paper by Hinton developed the model using thermal conductivity model approach ⁵. It uses general analysis by Biglari ⁶ that the sheared poloidal rotation

suppress the turbulent transport. In this study, the model is done using one-field approach. Later in the year 1992, Hinton and Staebler studied the model using two-field approach. Using slab geometry, the velocity shear can be expressed from force balance equation as



Fig. 1 S-curve bifurcation diagram⁴.

$$v'_{E} = \frac{c}{eB} \frac{\partial}{\partial x} \left(\frac{1}{n(x)} \right) \frac{\partial p(x)}{\partial x}.$$
 (1)

where $\frac{\partial p_i}{\partial r}$ is the pressure gradient, *B* is magnetic field, *n* is the density, *c* is speed of light and *e* is the elementary charge. Note that the constants will be normalized from now on. In 2008, Malkov and Diamond ⁴ studied this model using analytical approach in one dimension using slab geometry assumption. In their model, the heat transport equation is of the form:

$$\frac{\partial p}{\partial t} - \frac{\partial}{\partial x} \left[\chi_0 + \frac{\chi_1}{1 + \alpha v_E^{\prime 2}} \right] \frac{\partial p}{\partial x} = H(x) , \qquad (2)$$

where neoclassical transport coefficient χ_0 and anomalous transport coefficient χ_1 is represented as total transport of the system, *H* is the heat source, and α is the strength of the suppression term as shown in equation (1).

3. Analytical Study

At stationary state, the time derivative term in equation (2) is vanished, the stationary condition can be shown as

$$-\left[\chi_0 + \frac{\chi_1}{1 + \alpha v_E'^2}\right] \frac{\partial p}{\partial x} = \int_0^x H(x') dx' = Q(x), \qquad (3)$$

where Q is the heat flux of the source given into the plasma. The following assumption is used:

$$\mathbf{v}_E' = \left| \mathbf{p}' \right|,\tag{4}$$

So the function Q which represents heat flux can be viewed as a function of negative pressure gradient. This function can be solved analytically and the solution for bifurcation to exist, i.e. having both local maximum and minimum, is possible only when the following condition is satisfied:

$$\lambda = \frac{\chi_1}{\chi_0} > 8. \tag{5}$$

This analytical study implies that there exists a threshold for ratio of turbulent to neoclassical transport coefficients. This is a necessary condition for the existence of a bifurcation curve.

4. Numerical Results and Discussions

In this numerical study, constant source of heat is selected for the purpose of easy verification with analytical expectation, for example since

$$Q_{s}(x) = \int_{0}^{x} H_{s}(x') dx', \qquad (6)$$

the magnitude of heat source specified by user can be used to explore the transitional threshold at Q_{crit} . In running of these simulations, the constant values such as χ_{0} , χ_{1} , and α are selected to optimize the results according to analytical study previously, and also to make sure the ratio is high enough so *H*-mode can be obtained.

4.1 Case $Q_s < Q_{crit}$:

Fig. 2 shows numerical result of this bifurcation model when the heat flux Q_s is less than critical value Q_{crit} necessary for plasma to make a transition to H-mode. During steady state, the plasma cannot reach *H*-mode so it stays in *L*-branch only. Note that in the plot, the blue line represents analytical result, the red square dot represents numerical result one for each grid point, the left panel is initial state, and the right panel is the final state. The left plots show pressure (left) and pressure gradient (right) profiles as a function of plasma radius. The blue line represents initial state and the red line represents final state. It can be observed that there is no drastic change in pressure or pressure gradient profiles which represent no formation of transport barrier.



Fig. 2 Bifurcation result and Pressure and pressure gradient profile when $Q_s < Q_{crit}$





Fig. 3 Bifurcation result and Pressure and pressure gradient profile when $Q_s > Q_{crit}$

Fig. 3 shows numerical result when the heat flux Q_s is higher than critical value Q_{crit} . Therefore, at stationary state plasma make a transition to *H*-mode so some data points fall in *H*branch. The left plots show that transport barrier does form at the edge of the plasma with the sudden increase in pressure gradient. One other thing that can be observed is that even though the heat flux is increased only from 2.4 to around 3.2 magnitude, the center pressure is increased significantly from 8 in *L*-mode to about 55 magnitude in *H*-mode, about 7 times increment. This confirms experimental observation that the plasma performance like pressure or temperature can be increased by a large amount with formation of transport barrier.

One other effect that can be studied is the perturbation effect in the heat flux given into the system. This is done by giving the constant source with magnitude almost enough for the plasma to make a transition to H-mode, then putting the random noise into the source such that on average the magnitude remains less than Q_{erit} but sometimes it can go over the threshold. The results showed that the perturbation does not affect the plasma system nor make the transition possible.

5. Summary

An analytical study shows that two conditions are necessary for plasma to make an L-H transition; a ratio between turbulent over neoclassical transport coefficients reaches certain value

and a source heat flux into the system is higher than critical value for the local maximum. Numerical results have confirmed that a source heat flux is needed to surpass the critical value in order for plasma to make a transition and when it happens an edge transport barrier is formed as steep pressure gradient. At last, the perturbation of the heat flux into the system does not significantly affect the plasma that is the plasma cannot enter *H*-mode when the plasma is disturbed slightly.

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